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BIon configuration with fuzzy S^2 structure

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Abstract: Some configurations of general dielectric D2-brane with both electric and magnetic fields are investigated. We find two classical stable solutions describing a BIon configuration with S^2 structure and a cylindrical tube respectively. Both of them can be regarded as the blown-up objects of the Born-Infeld string. The dependence of the geometry of these configurations on the Born-Infeld fields is analyzed. Also we find that similar BIon configuration exists as a fuzzy object in a system with N D0-branes. Furthermore this configuration is demonstrated to be more stable than the usual dielectric spherical D2-brane.

Keywords: Superstrings and Heterotic Strings, D-branes, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

It is well known that the concept of D-brane [1] arises as the extended objects that open strings can end on. The worldvolume dynamics of D-brane, which can be derived as the low energy effective action of open string, is described by the Born-Infeld (BI) action [2]. Dynamically, under some circumstances, a lower-dimensional brane can be blown-up to another higher-dimensional brane. Inversely, the lower-dimensional brane can be considered as a partially collapsed version of the higher-dimensional brane. It is observed in [3] that a BI string can tunnel to a tubular D2-brane or nucleate to spheroidal bulge of D2-brane in constant RR field background. Another observation in [4] is that a similar non-trivial background can blow up a bunch of D0-brane into fuzzy 2-sphere, which is the well-known Myers effect. Furthermore, [5] suggests that string carrying D0-brane charge can be 'blown-up' into supertube supported against collapse by electric-magnetic angular momentum. The supertubes are collections of type IIA fundamental strings and D0-branes, which have been expanded to tubular 1/4-supersymmetric D2-branes by the addition of angular momentum.

Born-Infeld theory admits some finite energy static solutions. These solutions, called as BIons [6], are with pointlike sources like the fundamental strings. They play the important role in the theory of D-branes as the ends of string intersecting the brane when the effects of gravity are ignored [6, 7]. In particular, the BIon solution can be used to analyze the tunnelling of some D-brane configurations. Emparan [3] investigated some static unstable configurations in the presence of a constant 4-form RR field strength. These configurations include the bound configuration of a spherical brane and the BI string approaching the limit of a spike at large distance. This BI string possesses the character of BIon solution. According to the BIon description of solutions in Born-Infeld theory, we shall call the brane configuration with the BIon's character as the BIon configuration. In this viewpoint, the BIon configuration considered by Emparan is the bound configuration of a D2-brane and the fundamental strings which can be expressed as the sources of a Coulomb-like electric field. The BIon solution in [5] describes the configuration of the type IIA strings ending on the D2-brane and D0-branes. In order to preserve the supersymmetry, the 'dissolved' fundamental strings which can be interpreted as the sources of the electric field and the 'dissolved' D0-branes which are realized as the magnetic fluxes must be considered. So this BIon configuration is the bound brane configuration of a D2-brane, the fundamentle strings and the D0-branes. Recently, Hyakutake [8] found the fuzzy BIon solution by establishing the dual description of the BIon solution in [5]. The fuzzy BIon is described by the configuration obtained by pulling the fuzzy plane in a fixed direction. It is natural to ask whether there exist more general BIon solutions or not, and how the geometries of the brane configurations are affected by the BIon's properties. In this paper, we are interested in finding some new BIon solutions, and analyzing the geometric characteristics of the BIon configurations corresponding to them.

If we want to investigate more general BIon configurations, which include the spherical D2-brane, the effective BI action considered by us should include the Chern-Simons coupling term in the presence of non-zero RR field strength. The reason is due to the Myers effect [4] that the above non-trivial background can blow-up a collection of D0-branes into a fuzzy two-sphere. In the presence of the constant RR field strength, the matrix fields of N coincident D0-branes' configuration are no longer globally stable if they were diagonalized simultaneously. Also it is known that in the supertube of [5] the addition of D0-branes, of which the charge can be interpreted as the magnetic flux, is to provide a stabilization mechanism of preventing from collapse by crossed electric and magnetic fields. In order to make our considering BIon configuration stable, it should be the bound brane configurations of a D2-brane, fundamental strings and D0-branes in the background of the non-trivial RR field. Since the fundamental strings can be expressed as the sources of the electric field and the D0-brane charge can be interpreted as the magnetic flux, our starting point should be the effective BI action with both the electric field and the magnetic field in the presence of the constant RR field strength.

In the next section, we shall discuss some BIon configurations from the effective BI action as mentioned above. Indeed, We find two classical stable solutions describing the BIon configurations with S^2 structure and with tube structure respectively. The former, which can be regarded as the bound brane configuration of a spherical D2-brane, fundamental strings and D0-brane, is one of the main results in this paper. Subsequently, in terms of the dielectric effect from the viewpoint of N coincident D0-branes, we find that the BIon configuration with S^2 structure also exist as a fuzzy object. The energy of the BIon configuration with the fuzzy S^2 structure is lower than that of the configuration of the usual fuzzy S^2 found by Myers [4]. That is, the former is more stable that the latter.

2. BIon on a dielectric spherical D2-brane

As mentioned above, here we focus on a D2-brane with N magnetic fluxes in the presence of constant RR flux $F^4_{0123} = -4h$. For generality we also allow the electric field on the world volume, which can lead to the growing of a spike on the usual dielectric spherical D2-brane. The BI action including the Chern-Simons terms, which governs the low energy dynamics of D2-brane in a RR background, reads as

$$I = -T_2 \int d^3\xi \left\{ \sqrt{-\det(P[G]_{ab} + \lambda F_{ab})} \right\} + \mu_2 \int P[C^{(3)}] \,, \tag{2.1}$$

where $T_2 = 2\pi/(2\pi l_s)^3 g_s$ is the D2-brane tension and $\lambda = 2\pi l_s^2$. As usual $P[\cdots]$ is used to clarify the the pullbacks, and the potential $C^{(3)}$ of the R-R field is defined by $F^4 = dC^{(3)}$. Since we are interested in static configuration with axial symmetry, it is convenient to use the cylindrical coordinates

$$x^0 = t \,, \qquad x^1 = R(z)\cos\theta \,, \qquad x^2 = R(z)\sin\theta \,, \qquad x^3 = z \,, \tag{2.2}$$

where (t, θ, z) are the world volume coordinates. For time-independent magnetic field and electric field in z direction, the BI 2-form field strength can be expressed as

$$F = Edt \wedge dz + Bdz \wedge d\theta \,. \tag{2.3}$$

Under the above conditions the D2-brane action can be reduced to

$$I = -T_2 \int dt d\theta dz \left\{ \sqrt{R^2(-\lambda^2 E^2 + 1) + \lambda^2 B^2 + R^2 R_z^2} - 2hR^2 \right\}. \tag{2.4}$$

The hamiltonian of the system is defined as

$$\mathcal{H} \equiv \mathcal{D}E - \mathcal{L} \,. \tag{2.5}$$

Here $\mathcal{D}$ is the 'electric displacement' which is defined as $\mathcal{D} = \frac{1}{T_2\lambda}\frac{\partial L}{\partial E}$, and it obeys the Gauss law constraint $\partial_z \mathcal{D} = 0$. E can be express by using $\mathcal{D}$ as

$$E = \frac{\mathcal{D}}{\lambda R}\sqrt{\frac{R^2(1+R_z^2) + \lambda^2 B^2}{\mathcal{D}^2 + R^2}} \,. \tag{2.6}$$

So the hamiltonian density becomes

$$\mathcal{H} = T_2 \left\{ \frac{1}{R}\sqrt{(\mathcal{D}^2 + R^2)(R^2(1+R_z^2) + \lambda^2 B^2)} - 2hR^2 \right\}. \tag{2.7}$$

The Euler-Lagrange equation for A_x and A_θ can produce another constraint on the magnetic field. If we define $\Pi = -\frac{1}{T_2\lambda}\frac{\partial L}{\partial B}$, the constraint reads as $\partial_z \Pi = \partial_\theta \Pi = 0$. So Π is a constant. Furthermore, we can establish the relation between the magnetic field B and Π in the following form

$$B = \frac{\Pi R^2}{\lambda}\sqrt{\frac{1+R_z^2}{\mathcal{D}^2 + (1-\Pi^2)R^2}} \,. \tag{2.8}$$

It should be noticed from (2.8) that the number of the magnetic flux per unit area becomes infinitely small as R approach to zero, unlike the case with no electric field where it is uniform. Quantization of the magnetic flux requires

$$N = \frac{1}{2\pi}\int d\theta dz B = \frac{1}{2\pi\lambda}\int d\theta dz \sqrt{\frac{\Pi^2 R^4(1+R_z^2)}{\mathcal{D}^2 + R^2 - \Pi^2 R^2}} \,. \tag{2.9}$$

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The hamiltonian density can be rewritten in terms of Π as

$$\mathcal{H} = T_2 \left\{ (\mathcal{D}^2 + R^2) \sqrt{\frac{1 + R_z^2}{\mathcal{D}^2 + (1 - \Pi^2) R^2}} - 2hR^2 \right\}. \tag{2.10}$$

Since the negative h will oppose the expansion of the brane, we will assume $h \geq 0$ in the following.

It is instructive to first consider the tube-like solution with $R_z = 0$ for all z. Since the configuration is noncompact, we should assume that the length in the z direction is taken as L to make the energy finite. Using the constraint (2.9) we can eliminate Π in the (2.10) to give

$$\mathcal{H} = T_2 \left\{ \frac{1}{R} \sqrt{(R^2 + \Omega^2)(R^2 + \mathcal{D}^2)} - 2hR^2 \right\}, \tag{2.11}$$

where $\Omega = \lambda N/L$. It is easily seen that for nonzero Ω and $\mathcal{D}$, the BI-string has infinite energy, thus is unstable. It would be blown-up to tube configuration at local minimum of the energy. If $h = 0$ this is just the supertube solution. With h turned on, the tube should grow larger (figure 2b), which can be understood as the Myers effect. There is also a tube solution with larger radius identified with a sphaleron on top of the potential barrier (figure 2d). When h increases to some critical value, the stable tube would become completely unstable.

Now let us turn to general case with nonzero R_z and look for static solution by extremizing the energy with respect to the field R. The static equation for R, which is gotten by varying (2.10) while N is fixed, is

$$\frac{\partial \mathcal{H}}{\partial R} + \frac{\partial \mathcal{H}}{\partial \Pi} \frac{\partial \Pi}{\partial R} - \frac{d}{dz} \frac{\partial \mathcal{H}}{\partial R_z} = 0. \tag{2.12}$$

After some algebraic manipulation, it can be rewritten as (only valid for $R_z \neq 0$)

$$\frac{d}{dz} \left\{ \sqrt{\frac{\mathcal{D}^2 + (1 - \Pi^2) R^2}{1 + R_z^2}} - 2hR^2 \right\} = 0. \tag{2.13}$$

Introducing the integration constant C, we find

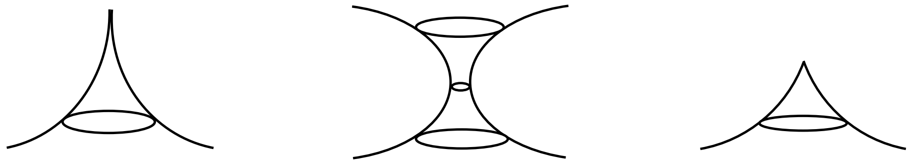
$$\begin{aligned} R_z &= \pm \frac{1}{2hR^2 + C} \sqrt{\mathcal{D}^2 + (1 - \Pi^2) R^2 - (2hR^2 + C)^2} \\ &= \pm \frac{2h}{2hR^2 + C} \sqrt{(R_+^2 - R^2)(R^2 - R_-^2)}. \end{aligned} \tag{2.14}$$

Here $R_\pm$ are defined as

$$R_\pm^2 = \frac{1}{8h^2} \left\{ (1 - \Pi^2 - 4hC) \pm \sqrt{(1 - \Pi^2 - 4hC)^2 + 16h^2(\mathcal{D}^2 - C^2)} \right\}. \tag{2.15}$$

We also rewrite the BI field strength for further convenience as

$$E = \frac{\mathcal{D}}{\lambda(2hR^2 + C)}, \qquad B = \frac{\Pi R^2}{\lambda(2hR^2 + C)}. \tag{2.16}$$



(a) (b) (c)

Figure 1: Static solutions for $h = 0$. (a) Non-singular BIon solution. (b) D-branes joined by a throat. (c) Singular BIon solution.

Though (2.14) can always be integrated in terms of elliptic integrals as done in [9], we have no need to do it for general case.

In order to gain some insight we shall have some brief discussion about the solution in the absence of RR fields. If we set $h = 0$, (2.14) becomes $R_z = \pm\frac{1}{C}\sqrt{(1-\Pi^2)R^2 + \mathcal{D}^2 - C^2}$. The equation for R can be easily integrated. The solution for the case $C = \mathcal{D}$ is $R = \exp\{\pm\frac{\sqrt{1-\Pi^2}}{\mathcal{D}}(z - z_0)\}$. This solution leads a logarithmic bending of D2-brane. The electric potential also admits a form of logarithm, while the magnetic field is proportional to R. These are the characteristics of the BIon solution. Actually It is just the BIon configuration preserving 1/4 supersymmetry found in [5]. The solution corresponding to $C^2 > \mathcal{D}^2$ is $R = \sqrt{\frac{C^2-\mathcal{D}^2}{1-\Pi^2}}\cosh\{\pm\frac{\sqrt{1-\Pi^2}}{C}(z - z_0)\}$. This is electric neutral catenoidal solution representing two D-branes joined by a throat. The case $C^2 < \mathcal{D}^2$ implies the solution $R = \sqrt{\frac{\mathcal{D}^2-C^2}{1-\Pi^2}}\sinh\{\pm\frac{\sqrt{1-\Pi^2}}{C}(z - z_0)\}$, which is a singular deformation of the BIon solution as discussed in [6]. These solutions can been seen in figure 1.

For h nonzero, we note that the term in the square root of (2.14) should be larger than zero, so R is restricted between R_- and R_+. To be more precisely R can change from R_- to R_+ or from R_+ to R_-. General solution extending in z direction can be composed through periodic continuation. If $R_- < 0$, i.e. $C^2 < \mathcal{D}^2$, there is a singularity at finite z as R approach zero. As expected from the case $h = 0$, this represents a spike with finite length. Only the case $R_- = 0$ admits an infinitely long spike without singularity. This is the main configuration we interested in throughout this section.

In this case we can integrate (2.14) easily to find

$$\sqrt{R_+^2 - R^2} + \frac{\mathcal{D}}{2hR_+}\ln\frac{R_+ + \sqrt{R_+^2 - R^2}}{R} = |z - z_0|\,, \tag{2.17}$$

with $R_+^2 = \frac{1-\Pi^2-4h\mathcal{D}}{4h^2}$. In fact, the solution (2.17) describes the BIon configuration. As R approach R_+ the second term can be neglected. This leads to $R^2 + |z - z_0|^2 = R_+^2$ which describes a sphere at $z = z_0$ with radius R_+. As R approach zero the second term is dominant and $R \propto \exp(-\frac{1}{L}(|z - z_0| - R_+))$. This is exactly the BIon solution satisfying the equation $dz = \pm\frac{L}{2}\frac{dR^2}{R^2}$, with $L_s = \frac{\mathcal{D}}{2hR_+}$ characterizing the length of the spike. Furthermore the BI field strength can be rewritten as

$$F = \frac{\mathcal{D}}{2\lambda hR_+R}dt\wedge dR + \frac{\Pi R}{2\lambda hR_+}dR\wedge d\theta. \tag{2.18}$$

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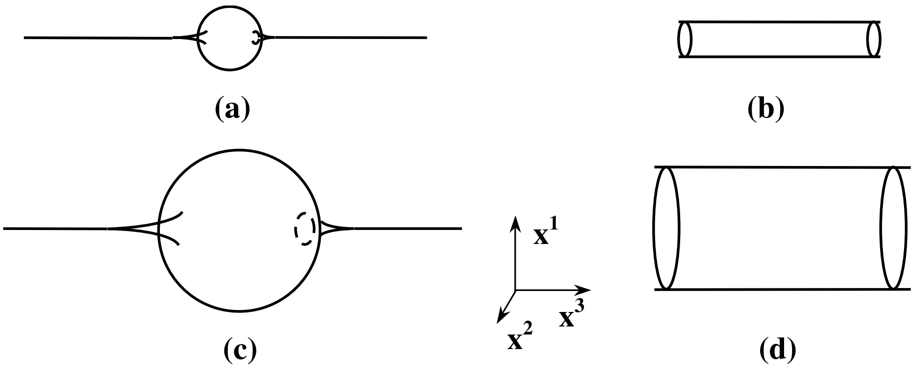

Figure 2: Static solutions for $h \neq 0$. Stable solution: (a) BIon configuration with a sphere, (b) Tube solution; (c) and (d) are the corresponding unstable configurations with larger dimension.

This suggest a radial Coulomb-like charge on the worldvolume, as expected from the BIon solution. So the whole solution representing a BIon configuration with a sphere, see figure $2a$ and$2c$. Similar configuration was discovered in [3] with only the electric field turned on, which is the sphalerons on top of a potential barrier. But as we shall see later, when magnetic field exist, this configuration would be stable.

The energy of the configuration (2.17) is given by

$$H = 2hT_2V_3 + T_0N\Pi + 2\pi T_2\mathcal{D}\int dz\,, \tag{2.19}$$

where $V_3 = \frac{4}{3}\pi R^3$ is the volume of the sphere. As shown in [7] the quantization condition on the $\mathcal{D}$-flux is $\frac{1}{2\pi}\int d\theta\mathcal{D} = ng$, i.e., $\mathcal{D} = ng$. By using this relation the last term in (2.19) can be written as $nT_f\int dz$ which is just the energy of n fundamental string. So one would observe that the energy exactly split into three parts arising from the energy of RR flux, D0-branes and fundamental strings. So we can identify this configuration as the bound state of D2-brane with N D0-branes and n fundamental strings dissolved in the worldvolume. Furthermore it is noticed that the second term in (2.19) is not the pure energy of N D0-branes, but includes the contribution of $\mathcal{D}$ through Π. This can be explained by considering the interaction of the fundamental strings and D0-branes. The N units magnetic fluxes can be written as

$$N = \frac{\Pi}{h\lambda}R_+ = \frac{\Pi\sqrt{\Lambda^2 - \Pi^2}}{2h^2\lambda}\,, \tag{2.20}$$

from which we can express Π^2 as a function of N

$$\Pi^2 = \frac{1}{2}\left(\Lambda \pm \sqrt{\Lambda^2 - 4\Xi^2}\right). \tag{2.21}$$

Here $\Lambda^2 \equiv 1 - 4h\mathcal{D}$ and $\Xi \equiv 2h^2\lambda N$. We consider the case with small BI field that $\Xi \ll 1$ and $\Xi \ll \Lambda$, which is appropriate for our discussion below and for later comparison with D0-brane description. The two possible value of Π^2 can be approximated as $\Pi_1^2 = \Lambda^2 - \frac{\Xi^2}{\Lambda^2}$ and $\Pi_2^2 = \frac{\Xi^2}{\Lambda^2}$. Inserting them back in (2.19) we can get the corresponding energy (below

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we have neglected the energy of the BI-string since it is always the same in each case)

$$H = \begin{cases} \dfrac{NT_0}{\Lambda}\left(\Lambda^2 - \dfrac{\Xi^2}{6\Lambda^2}\right) & R_1 = \dfrac{\Xi}{2h\Lambda} \\ \dfrac{1}{3}NT_0\dfrac{\Lambda^3}{\Xi} & R_2 = \dfrac{\Lambda}{2h} \end{cases} \tag{2.22}$$

The situation with only magnetic or electric field had been explored extensively. Now we shall investigate what will happen when we turn on both fields. First with no magnetic fluxes, there exist two configurations with $R_1 = 0$ and $R_2 = \frac{1}{2h}\sqrt{1 - 4h\mathcal{D}}$. They are just the stable BI-string and the S^2 configuration with string ending as discussed in [3]. As we turn on the magnetic fluxes, we have two configuration with string ending on S^2, which is different from the case with only electric field. One of them is with a smaller sphere of which the radius is $R_1 = \frac{\Xi}{2h\Lambda}$, and the other is with a larger sphere of which the radius is $R_2 = \frac{\Lambda}{2h}$. It can be seen from (2.22) that the former have lower energy than the latter under the condition of small BI field, i.e., the former is more stable than the latter. Furthermore, one can see that under the same circumstance the energy of the latter configuration is independent of the number of magnetic flux N. Hence, the configuration with a larger sphere corresponds to the S^2 configuration with string ending in the case of no magnetic field, and it is unstable. As discussed previously now BI-string would gain an infinite energy shift and become unstable. However, because the magnetic flux can be interpreted as a 'dissolved' D0-brane charge, turning on the magnetic field implies that some D0-branes is added to the system. Due to the Myers effect, a stuck of D0-branes is blown-up a sphere. This makes the BIon configuration with string ending on the smaller sphere become stable. On the other words, as N being turned off the bludge shrinks to zero size and the BI-string is recovered. Similarly, we can also start from the case with only magnetic field, where there is a stable S^2 known as the dielectric sphere. We know that in type IIA superstring theory there exists the BIon configuration which represents a fundamental string ending on a bound of a D2-brane and D0-branes [5]. From the viewpoint of the worldvolume theory on the D2-brane, the fundamental string is expressed as a source of a Coulomb-like electric field. Thus, as we turn on electric field, the fundamental string is effectively attached to the spherical brane configuration. Conclusively, the stable configuration is the spherical brane configuration with spikes.

3. Fuzzy BIon from dielectric D0-branes

In this section, we shall show that the configuration we have found in the previous section also exist as a bound state of N D0-branes. We start with the low energy effective action of N D0-branes, which is the non-abelian extension of BI action proposed in [4], the leading nontrivial terms are

$$S \sim -NT_0V_1 + \frac{1}{4}\lambda^2 T_0 \int dt\, \mathrm{Tr}(2D_0\phi^i D_0\phi^i + [\phi^i, \phi^j]^2) - \frac{4}{3}ih\lambda^2\mu_0\epsilon_{ijk}\int dt\, \mathrm{Tr}(\phi^i\phi^j\phi^k), \tag{3.1}$$

where $D_0\phi^i = \partial_0\phi^i + i[A_0, \phi^i]$ and $i, j, k = 1, 2, 3$. We have consistently set $\phi^4, \ldots, \phi^9$ to zero, and the RR background $F^4_{0123} = -4h$ has been introduced into the action through

Chern-Simons term. It should be noticed that this action is valid only when the commutators $i\lambda[\phi^i,\phi^j]$ are small enough, or $h^2\lambda N \ll 1$. The corresponding lagrangian can be written as

$$L = -NT_0 + \lambda^2 T_0 \,\mathrm{Tr}\left\{\frac{1}{2}D_0\phi^i D_0\phi^i + \frac{1}{4}[\phi^i,\phi^j]^2 - i\alpha\epsilon_{ijk}\phi^i\phi^j\phi^k\right\}. \tag{3.2}$$

Here we define $\alpha = \frac{4}{3}h$ for further convenience. The equation of motion for ϕ^i and Gauss law constraint derived directly from the lagrangian are

$$-\,[D_0,[D_0,\phi^i]] + \frac{3i\alpha}{2}\epsilon_{ijk}[\phi^j,\phi^k] + \Big[[\phi^i,\phi^k],\phi^k\Big] = 0\,, \qquad [\phi^i,[D_0,\phi^i]] = 0\,. \tag{3.3}$$

Following [10], for configurations with axial symmetry, we choose the ansatz

$$\begin{aligned}
\phi^1_{mn} &= \frac{1}{2}\rho_{m+1/2}\delta_{m+1,n} + \frac{1}{2}\rho_{m-1/2}\delta_{m,n+1}\,,\\
\phi^2_{mn} &= \frac{i}{2}\rho_{m+1/2}\delta_{m+1,n} - \frac{i}{2}\rho_{m-1/2}\delta_{m,n+1}\,,\\
\phi^3_{mn} &= z_m\delta_{m,n}, \qquad A_{0mn} = a_m\delta_{m,n}\,.
\end{aligned}$$

Here we only consider the static configuration. Thus, substituting the ansatz into (3.3), we get

$$\begin{aligned}
(a_{m+1}-a_m)^2 - (z_{m+1}-z_m)^2 + 3\alpha(z_{m+1}-z_m) + \frac{1}{2}\left[\rho^2_{m-1/2} - 2\rho^2_{m+1/2} + \rho^2_{m+3/2}\right] &= 0\,,\\
-\frac{3\alpha}{2}(\rho^2_{m-1/2} - \rho^2_{m+1/2}) - \left[\rho^2_{m+1/2}(z_{m+1}-z_m) - \rho^2_{m-1/2}(z_m - z_{m-1})\right] &= 0\,,\\
\rho^2_{m+1/2}(a_{m+1}-a_m) - \rho^2_{m-1/2}(a_m - a_{m-1}) = 0\,.
\end{aligned} \tag{3.4}$$

To solve the above equations, we first choose the ansatz $\rho_{m+1/2} = (3/2)\,\alpha \times \times\sqrt{(j+m+1)(j-m)}$, which is reasonable since ϕ^i just become the fuzzy sphere solution found in [4] when $z_m = \frac{3}{2}\alpha m$. Now we are to find general z_m and a_m as a solution of (3.4). Substituting $\rho_{m+1/2}$ into the second equation of (3.4) yields

$$z_{m+1} - z_m = \frac{3}{2}\alpha + \frac{C}{\rho^2_{m+1/2}}\,. \tag{3.5}$$

Then inserting this back into the first equation in (3.4), one finds

$$a_{m+1} - a_m = \pm\frac{C}{\rho^2_{m+1/2}}\,. \tag{3.6}$$

It can be easily checked that this solution also satisfies the third equation in (3.4). Here C is an arbitrary constant. For our convenience we can set $z_0 = a_0 = 0$. From (3.5) and (3.6) one can find

$$\begin{aligned}
z_m &= \frac{3}{2}\alpha m + \mathrm{sgn}(m)\sum_{k=0}^{|m|-1}\frac{C}{\rho^2_{m+1/2}}\,,\\
a_m &= \pm\mathrm{sgn}(m)\sum_{k=0}^{|m|-1}\frac{C}{\rho^2_{m+1/2}}\,.
\end{aligned} \tag{3.7}$$

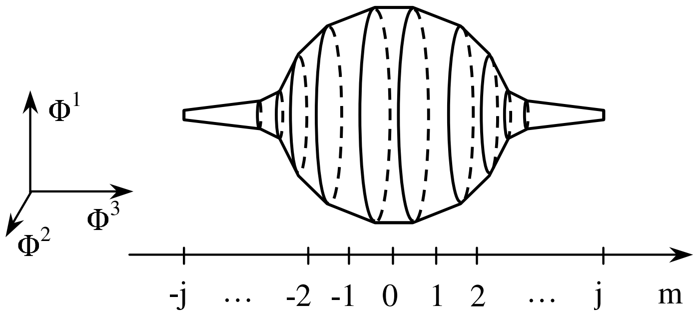

Figure 3: Fuzzy BIon with S^2 structure as static solutions of dielectric D0-branes.

The algebra describing the fuzzy configuration can be written as

$$[\phi^1, \phi^2] = i\frac{3}{2}\alpha(\phi^3 \mp A_0)\,, \qquad [\phi^2, \phi^3] = i\frac{3}{2}\alpha\phi^1 \pm [\phi^2, A_0]\,, \qquad [\phi^3, \phi^1] = i\frac{3}{2}\alpha\phi^2 \pm [A_0, \phi^1]\,. \tag{3.8}$$

If we have redefinitions $\phi^1 \equiv \tilde{\phi^1}$, $\phi^2 \equiv \tilde{\phi^2}$ and $\phi^3 \equiv \tilde{\phi^3} \pm A_0$, (3.8) can be rewritten in terms of $\tilde{\phi^i}$ as $[\tilde{\phi^i}, \tilde{\phi^j}] = i\frac{3}{2}\alpha\epsilon_{ijk}\tilde{\phi^k}$. This is essentially the $SU(2)$ algebra with a deformation in the z direction corresponding to two lumps protruding in the north and south pole of fuzzy S^2. The shape of the lump is completely determined by A_0. For the bound brane configuration including the string, which we are just interested in, A_0 can not be set to zero. The reason is that A_0 is related to the string charge as explained in [11].

When C equals to zero in (3.7), we have $\phi^3 \equiv \tilde{\phi^3}$. And it is obvious that ϕ^i satisfy the $SU(2)$ algebra representing a fuzzy sphere of radius $h\lambda N$. Thus we recover the fuzzy S^2 blown-up due to Myers effect. But the solution is more interesting with nonzero C. The non-vanishing contribution of the lump is described by

$$\Delta z_m = -\frac{1}{2}\frac{C}{4mh^2}\frac{\Delta\rho_m^2}{\rho_{m+1/2}^2}\,. \tag{3.9}$$

Here we use the identity $\Delta\rho_m^2 = -8mh^2$. This is just the regularized version of the differential equation that BIon should satisfy. The coefficient $L_s \equiv \frac{C}{4mh^2}$ is identified with the length of the spike we defined previously. In the spike-dominant region $m \sim N/2$, the length is approximate to $\frac{\lambda C}{\Xi}$. It is easily seen that the length is inverse proportional to the Ξ, which agree with previous result implying the spike would be suppressed by the magnetic flux. Furthermore, if we identify λC with $\mathcal{D}$, the radius of the sphere and length of the spike are given by $R = h\lambda N$ and $L_s = \frac{\mathcal{D}}{\Xi}$ respectively, which is completely the same as the previous results for small $\mathcal{D}$. Thus we find a fuzzy counterpart of the BIon configuration with a sphere given before. This fuzzy object can be seen as jointed by many fuzzy segments in different positions of z direction, see figure 3, with $\rho_{m+1/2}$ giving the radius of the joint between mth and $(m+1)$th segments. If we take the continuous limit $N \to \infty$ keeping $h\lambda N$ fixed, the fuzziness(or the area of each segment) vanish and we get a smooth object.

In order to analyze the stability of our solution, we should calculate the correspondent energy. The hamiltonian can be derived from (3.2) as

$$H = NT_0 + \lambda^2 T_0 \,\mathrm{Tr}\left(\frac{1}{2}\partial_0\phi^i\partial_0\phi^i + \frac{1}{2}[A_0,\phi^i]^2 - \frac{1}{4}[\phi^i,\phi^j]^2 + i\alpha\epsilon_{ijk}\phi^i\phi^j\phi^k\right). \tag{3.10}$$

Imposing the static condition and using (3.3) to eliminate the term with A_0 yields

$$H = NT_0 - \frac{1}{4}\lambda^2 T_0 \,\mathrm{Tr}\left(i\alpha\epsilon_{ijk}\phi^i[\phi^j,\phi^k] - [\phi^i,\phi^j]^2\right). \tag{3.11}$$

Substituting the solution (3.7) into the hamiltonian, we find

$$\begin{aligned} H &= H_0 - \frac{1}{2}\lambda^2 T_0 C \sum_{m=-j}^{j}\left(2\alpha - 4h^2\alpha|m|\sum_{k=0}^{|m|-1}\frac{1}{\rho^2_{k+1/2}} + \frac{C}{\rho^2_{m+1/2}}\right) \\ &= H_0 - \frac{1}{2}\lambda^2 T_0 C \sum_{m=-j}^{j}\Delta z_m - \frac{1}{3}h\lambda^2 T_0 C\,. \end{aligned} \tag{3.12}$$

Here H_0 is the energy of the usual fuzzy sphere and equals to $NT_0 - \frac{2}{3}NT_0(h^2\lambda N)^2(1-\frac{1}{N^2})$. It is interesting to observe that $\sum_{m=-j}^{j}\Delta z_m$ corresponds to the length of a string through the fuzzy sphere. The contribution to energy due to this string is similar to the that of fundamental string except that it is negative. If the radius $h\lambda N$ is fixed, the last term is inverse proportional to N and vanish in the large N limit. Thus the total energy is smaller than H_0 for $C > 0$, which means that our solution with spikes has lower energy than the fuzzy S^2. This implies that the dielectric sphere is not stable and can grow spikes to lower its energy if we allow A_0 to be nonzero. Conclusively, the BIon configuration given by us is more stable than the configuration of the fuzzy sphere done by Myers [4].

So from above analysis we can see that our solution of the N D0-brane action represent a BIon configuration with fuzzy S^2 structure, Which in the large N limit gives the same configuration found in the previous section when we analyze the D2-brane worldvolume action. This is what we have expected, and it also supply a new evidence for the equivalence of the two pictures.

4. Conclusions

Based on the physical consideration, here we have investigated the D2-brane in uniform RR background while we have incorporate both electric and magnetic field in the world volume of the brane. Some new static solutions of this system have been found. One among them is geometrically described by a two-sphere with a spike, charactered by the BIon of string ending on the spherical D2-brane. In the language of BIon configuration, it is a BIon configuration composed of the bound brane of a D2-brane, n fundamental strings and N D0-branes. By product, we also obtain a tube solution. They can be understood as the blown-up objects of the BI-string due to the Myers effect if we turn on the magnetic field. It has been shown that these configurations are classically stable. Due to the presence of the RR-flux, the configurations found above are quantum mechanically unstable, they would tunnel to configurations with larger radius. Further work is needed to elucidate the process through the construction of bounce solution and calculation of the decay rate.

In order to investigate the dual description of the dielectric D2-brane in the presence of electric field, we have also analyzed a system of N D0-branes in the same RR background. We give a solution describing a BIons configuration with fuzzy S^2 structure which is the same as in the D2-brane picture. By the redefinition of the matrix field, it is shown that algebra of the fuzzy configuration is a deformed SU(2) algebra which has nontrivial topology. By comparing the energy of this BIon configuration with that of the Myers' fuzzy S^2 configuration, we find that our configuration is more stable than Myers'.

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